Applications in recently published papers

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Dynamic Real-time Deformations using Space and Time Adaptive Sampling

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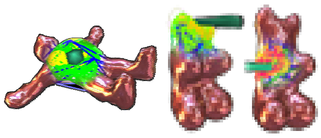
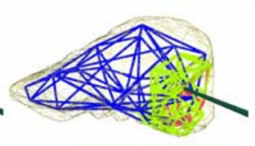


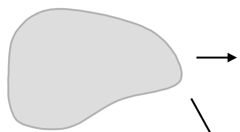
Surface and its Internal Model

Displayed surface
~ 10,000 triangles
30Hz

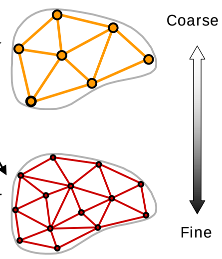


Internal physical model
~ 100 points
~ 1000Hz

Local Adaptive Refinement



- **Tetrahedral meshes**
  - Non-nested
  - Independent from each other
  - Optimized quality



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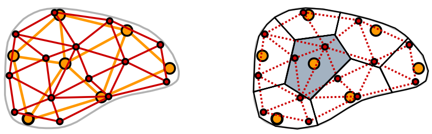
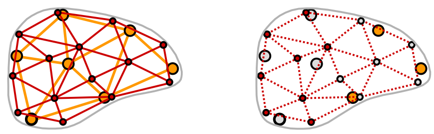
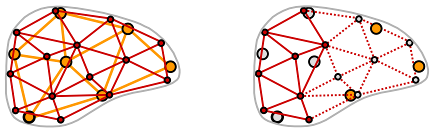


Local Adaptive Refinement



Local Adaptive Refinement



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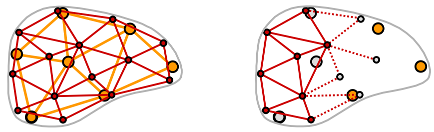

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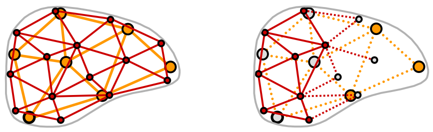
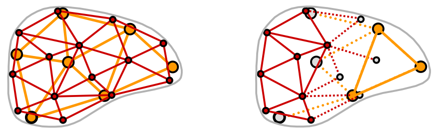
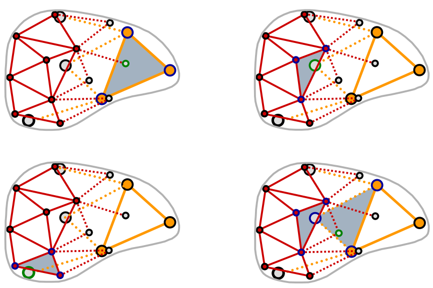


Local Adaptive Refinement

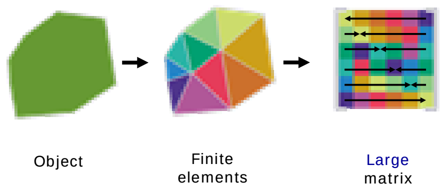


Local Adaptive Refinement



Space and Time Adaptive Model

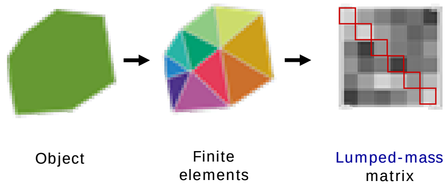
- **Finite element method**
  - + accuracy - speed



Space and Time Adaptive Model

- **Lumped mass formulation** **[O'Brien99], [Cotin00]**
  - accuracy + speed

Green strain tensor, strain rate tensor → $\mathbf{f}_i = m_i \mathbf{a}_i$



Space and Time Adaptive Model

- **Adapting space resolution**

$$\gamma = \Delta \mathbf{d} h^2 \approx \frac{\mathbf{f} h^2}{\mu V} = \frac{\mathbf{f}}{\mu h}$$

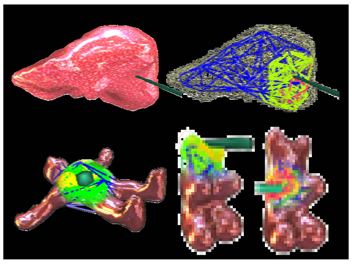
Lame constant for rigidity
Minimum distance between nodes
Laplacian of the displacement field

- **Adaptive time stepping**
  - Courant condition (CFL)

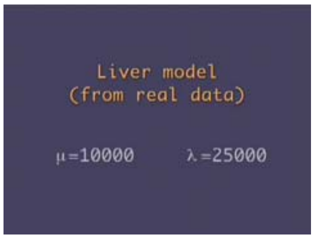
$$dt < h\sqrt{\frac{\rho_0}{2\mu + \lambda}}$$

Rest density
Lame constant for compressibility

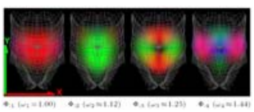
Results



Results



DyRT: Dynamic Response Textures for Real Time Deformation Simulation with Graphics Hardware



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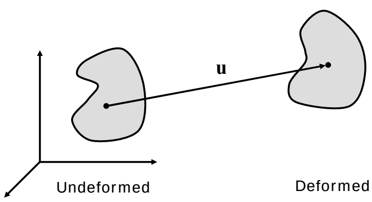
Modal Analysis: Overview

- **Linear elastodynamic equation for FEM**

$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{F}$ 3n x 3n matrix, 3n x 1 vector



Modal Analysis: Overview

- Linear elastodynamic equation for FEM

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{F}$$ 3n x 3n matrix, 3n x 1 vector

Modal transformation $\mathbf{u}(t) = \mathbf{\Phi}\,\mathbf{q}(t)$ ← $\mathbf{M}\,\mathbf{\Phi}_i = \lambda_i\,\mathbf{K}\,\mathbf{\Phi}_i$

generalized eigenvalue problem

$$\mathbf{M}_q\ddot{\mathbf{q}} + \mathbf{C}_q\dot{\mathbf{q}} + \mathbf{K}_q\mathbf{q} = \mathbf{Q} \Rightarrow m_i\ddot{q}_i + c_i\dot{q}_i + k_i q_i = Q_i$$

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$$

System of decoupled ODEs

Modal Analysis: Overview

- Linear elastodynamic equation for FEM

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{F} \qquad q_i^k = \alpha_i q_i^{k-1} + \beta_i q_i^{k-2} + \gamma_i Q_i^{k-1}$$

Modal transformation $\mathbf{u}(t) = \mathbf{\Phi}\,\mathbf{q}(t)$

IIR digital filter mode i, time k

$$\mathbf{M}_q\ddot{\mathbf{q}} + \mathbf{C}_q\dot{\mathbf{q}} + \mathbf{K}_q\mathbf{q} = \mathbf{Q} \Rightarrow m_i\ddot{q}_i + c_i\dot{q}_i + k_i q_i = Q_i$$

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$$

System of decoupled ODEs

Mode Shape & Amplitude

$$\mathbf{u}(t) = \mathbf{\Phi}\,\mathbf{q}(t) = \mathbf{\Phi}_i\,q_i(t)$$

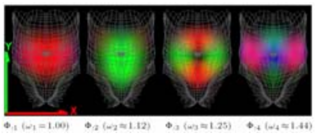
mode shape
mode amplitude

Modal Truncation

- Exploit dominant low frequency mode shapes only!
  - Higher modes heavily damped and die out fast.

$$\mathbf{u}(t) = \mathbf{\Phi}\,\mathbf{q}(t) = \mathbf{\Phi}_i\,q_i(t)$$

$\mathbf{u} = \mathbf{\Phi}\,\mathbf{q}$



$\mathbf{u} = \mathbf{\Phi}\,\mathbf{q}$ (m)

Exciting Modes with Rigid Motions

- Modal forcing term acting on a body point

accelerations $(\boldsymbol{\psi}^{k-1} - \boldsymbol{\psi}^{k-2})$

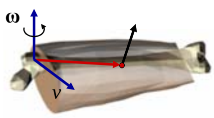
inertial forces $\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}^{k-1}$

$$\mathbf{q}^k = \boldsymbol{\alpha}\mathbf{q}^{k-1} + \boldsymbol{\beta}\mathbf{q}^{k-2} + \boldsymbol{\gamma}\mathbf{Q}^{k-1}$$

modal force

spatial velocity

$$\boldsymbol{\psi} = [\boldsymbol{\omega}^{\mathrm{T}} \quad \mathbf{v}^{\mathrm{T}}]^{\mathrm{T}}$$



Exciting Modes with Rigid Motions

- Modal forcing term acting on a body point

accelerations $(\boldsymbol{\psi}^{k-1} - \boldsymbol{\psi}^{k-2})$

m $\mathbf{Q} = \mathbf{H}$ (6)

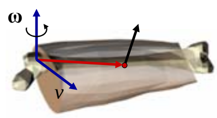
rigid motion transfer matrix

$$\mathbf{q}^k = \boldsymbol{\alpha}\mathbf{q}^{k-1} + \boldsymbol{\beta}\mathbf{q}^{k-2} + \boldsymbol{\gamma}\mathbf{Q}^{k-1}$$

modal force

spatial velocity

$$\boldsymbol{\psi} = [\boldsymbol{\omega}^{\mathrm{T}} \quad \mathbf{v}^{\mathrm{T}}]^{\mathrm{T}}$$



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Modal transformation $\mathbf{u}(t)=\mathbf{\Phi}\,\mathbf{q}(t)$

IIR digital filter
mode i, time k

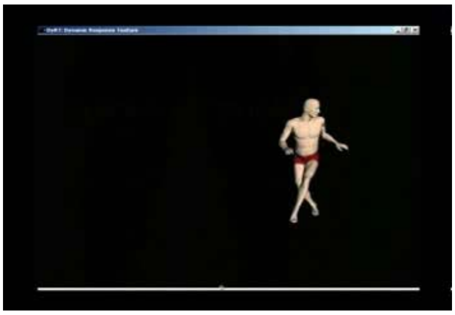
$$\mathbf{M_q}\ddot{\mathbf{q}}+\mathbf{C_q}\dot{\mathbf{q}}+\mathbf{K_q}\mathbf{q}=\mathbf{Q} \Rightarrow m_i\ddot{q}_i+c_i\dot{q}_i+k_i q_i=Q_i$$

$$\mathbf{C}=\alpha\mathbf{M}+\beta\mathbf{K}$$

System of decoupled ODEs

Results

Results



Interactive Skeleton-Driven Dynamic Deformations

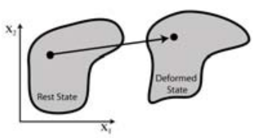
Steve Capell
Seth Green
Brian Curles
Tom Duchamp
Zoran Popovic

Univ. Washington

Elastic Solids

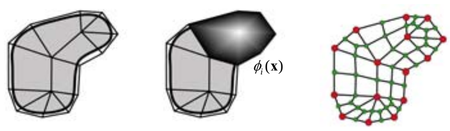
- Displacements to describe deformation

$$\mathbf{u}(\mathbf{x},t)=\sum_i \mathbf{q}_i(t)\phi_i(\mathbf{x})$$



Basis Functions

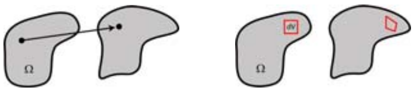
- Hierarchical basis functions



Governing Equations

- Euler-Lagrange equations

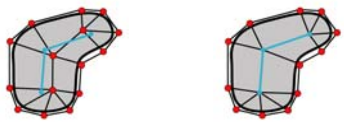
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\mathbf{q}}_i}\right)+\frac{\partial U}{\partial \mathbf{q}_i}=\mathbf{F}_i$$



$$T=\int_\Omega \frac{1}{2}\rho\|\dot{\mathbf{u}}\|^2 d\Omega \qquad U=\int_\Omega G\left(\frac{\nu}{1-\nu}\mathrm{tr}^2(\boldsymbol{\varepsilon})+\|\boldsymbol{\varepsilon}\|^2\right)d\Omega$$

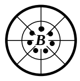
Skeletal Constraints

- Restrict the bones to along edges in the control lattice



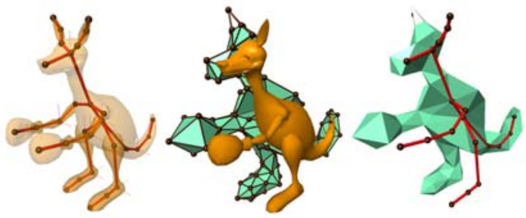
difficult to satisfy constraints easy to satisfy constraints

- Twisting constraints



$$U_{\text{twist}}=k\int_B\|\mathbf{u}\|^2 d\Omega$$

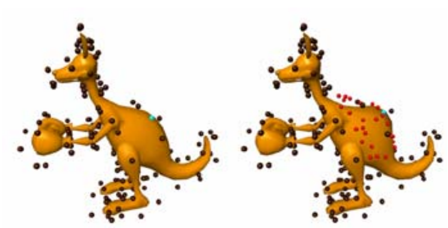
Instrument Models



Linear Strain & Blended Linear Strain

Adaptation

- Introduce finer basis functions for large deformation
- Remove basis functions for little deformation



Results